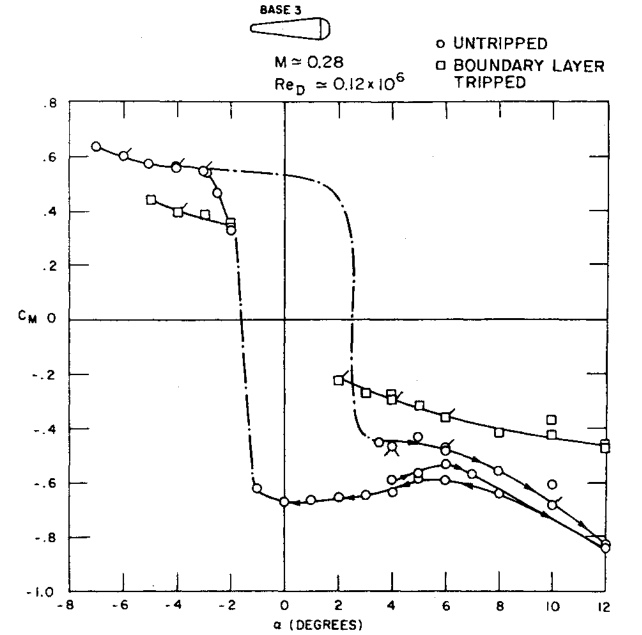

Fig. 6 Effect of boundary-layer trip on pitching moment coefficient vs angle of attack without sting.

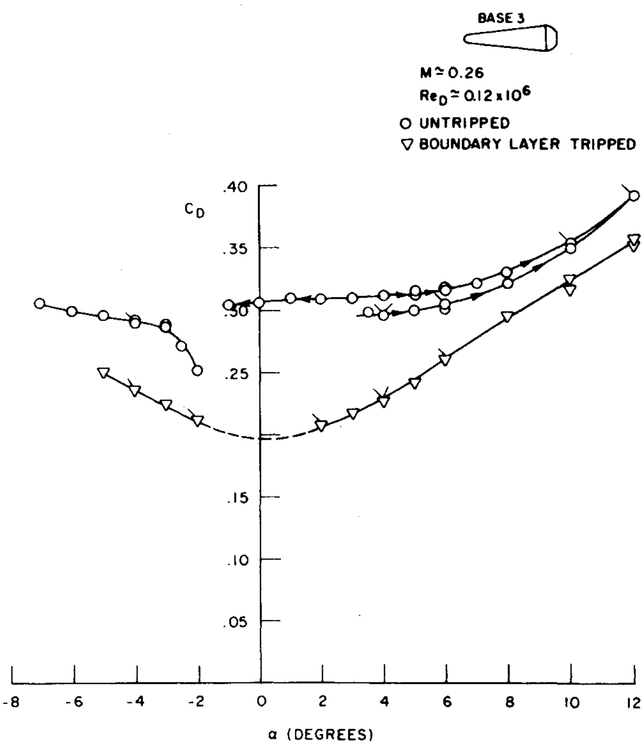

Fig. 7 Effect of boundary-layer trip on drag coefficient vs angle of attack without sting.

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Matching Procedure for Viscous-Inviscid Interactive Calculations

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Introduction

IN this Note, we describe a procedure for coupling a viscous boundary-layer calculation with a solution of the Euler equations. The interaction between the viscous calculation and the rotational inviscid calculation is accomplished by means of an iterative process. An iterative approach to the subsonic interaction problem is not uncommon; several researchers have used this approach in conjunction with the displacement thickness concept[1] to obtain better approximations to flows (e.g., Refs. 2 and 3). However, the present interactive method differs from these procedures in two ways. First, the present method does not rely solely on the mechanism of a physical displacement of the outer flow streamlines by the viscous layer to achieve coupling of the viscous and inviscid calculations. The interaction takes the form of an injection at solid surfaces, but it is different from the usual equivalent source distribution technique in that this injection has a momentum and enthalpy character. Second, the viscous solution is constructed in a manner suggested by the theory of matched asymptotic expansions. In order to illustrate the operation of this procedure more clearly, we discuss the specific case where the inviscid solution is an explicit time-marching, finite-difference calculation (e.g., MacCormack's method[4]). However, the applicability of the method is not restricted to this choice.

Interactive Procedure

Consider a portion of the flowfield in the neighborhood of an impermeable wall (Fig. 1). The Euler equations for steady flow may be written in the vector form:

$$\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0 \tag{1}$$

where

$$F=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ u(e+p)\end{bmatrix}\qquad G=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ v(e+p)\end{bmatrix}$$

Also, the steady Navier-Stokes equations may be written in the vector form:

$$\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}=0 \tag{2}$$

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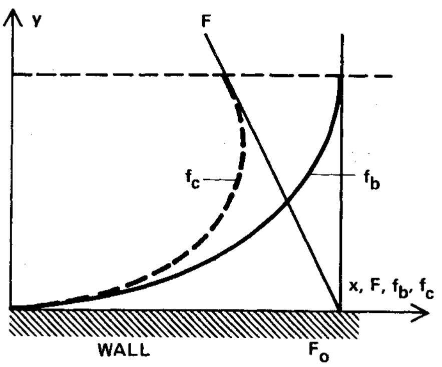

Fig. 1 Inviscid, boundary-layer, and composite solutions.

where f and g are four component vectors (see, for example, Ref. 4).

If only an inviscid solution for the flow was required, this could be achieved using the algorithm of Ref. 4. To accomplish this solution, one might specify boundary conditions on all four components of the vector G at the wall. The impermeability of the wall gives three conditions, since the vector components $G1$, $G2$, and $G4$ are identically zero at this location. (We affix a number to a vector to indicate the position of a component within that vector; for example, $G2$ is ρuv.) A specification remains for the component $G3$, which reduces to the surface pressure at this location, and this must be obtained in some other way. For example, the method of characteristics may be used to obtain a compatibility relation which gives the pressure at the wall.[5] The interactive procedure which we now describe provides replacements for the wall conditions, $G1$, $G2$, $G4=0$, for subsequent inviscid calculations within a viscous-inviscid iterative procedure.

An exact representation of the flow through the region shown in Fig. 1 is given by a solution of the Navier-Stokes equations. Let f and g be the vectors constructed from this solution. Also, we suppose that some solution of the Euler equations will provide a close approximation of the exact solution when $y>\delta$, and let F and G be the vectors constructed from this inviscid solution. Having identified the vectors f,g,F, and G with these two solutions, Eqs. (1) and (2) may be integrated from $y=0$ to $y=\delta$ to give:

$$G_\delta - G_0 = -\frac{\partial}{\partial x}\int_0^\delta F\,\mathrm{d}y \tag{3}$$

$$g_\delta - g_0 = -\frac{\partial}{\partial x}\int_0^\delta f\,\mathrm{d}y \tag{4}$$

where the subscripts indicate evaluation at the limits of integration. Since the two solutions may be taken to coincide for $y>\delta$, we specify $G_\delta = g_\delta$. Equations (3) and (4) then can be combined to give

$$G_0 = g_0 + \frac{\partial}{\partial x}\int_0^\delta (F-f)\,\mathrm{d}y \tag{5}$$

Equation (5), which relates the two hypothetical solutions, will serve as a starting point for our discussion of the solution technique in the viscous layer.

It is not our intention to solve the Navier-Stokes equations; therefore, we seek a suitable approximation of f and g on the interval $0 \le y \le \delta$. We represent the exact solution by a composite function f_c, where

$$f_c = F + f_b - F_0 \tag{6}$$

These solutions are shown in Fig. 1; f_b corresponds to a boundary-layer solution which uses, as boundary conditions, values from the inviscid solution at $y=0$. The composite function f_c is constructed in the spirit of a matched asymptotic expansion. The function f_c was chosen as an approximation of the exact solution for two reasons. First, we expect that f_c is a better representation of the exact solution than the usual boundary-layer solution, especially when there exist appreciable gradients in the y direction in the inviscid solution. Second, we employ f_c becuse it has computational advantages within the context of the iterative procedure, which are described later.

Applying Eq. (6) to Eq. (5) gives

$$G_0 = (g_b)_0 + \frac{\partial}{\partial x}\int_0^\delta (F_0 - f_b)\,\mathrm{d}y \tag{7}$$

Equation (7) can be used as the basis for an iterative solution technique in the following way:

1) $(G1)_0$, $(G2)_0$, and $(G4)_0$ are initially set to zero, and an inviscid solution is carried out.

2) Using inviscid values at $y=0$, a boundary-layer solution is performed.

3) Using values obtained from the inviscid and boundary-layer solutions, Eq. (7) is solved for new values of $(G1)_0$, $(G2)_0$, and $(G4)_0$. The vector component $(G3)_0$, which contains the surface pressure, is obtained through other considerations. Since Eq. (7) requires only surface values from the inviscid solution (a result of the choice of f_c), the numerical evaluation of the integral in this equation is not difficult, even in the general situation where the coordinate systems for the viscous and inviscid calculations are different. (It should be noted that the region shown in Fig. 1, which has been covered with a Cartesian coordinate system aligned with the wall, may be viewed as a small portion of a flow past a curved surface.)

4) Using new values of $(G1)_0$, $(G2)_0$, and $(G4)_0$, an inviscid solution is carried out.

5) Steps 2-4 are repeated until an acceptable degree of convergence is obtained.

Discussion

The interaction model which has been described here can be used conveniently with those numerical algorithms currently employed to solve the Euler equations in primitive variable form (e.g., Refs. 4 and 6). There is an alternative method for dealing with the numerical viscous-inviscid interaction when the inviscid flow is rotational (that is, the displacement thickness approach), but it is not conveniently used in problems involving complicated geometries. In such an approach, bodies are physically thickened, and it would be necessary to recompute the geometry of the problem at each step in the iteration. In the present method, the geometry of the solid surface must be dealt with only once, and remains unchanged throughout the iterative process. The equivalent source method, which has similar advantages over the displacement thickness approach for the case on an outer potential flow, appears as the first equation in Eq. (7) when the composite function (f_c) and the boundary-layer solution (f_b) are indistinguishable.

The present interactive procedure has been used as the basis for a numerical calculation of flow through a cascade of mildly curved compressor blades (see Ref. 7). In this calculation, the viscous-inviscid iterative scheme appeared to converge successfully, and without assistance, in the absence of boundary-layer separation.

Acknowledgment

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Dynamics of a Fluid Conveying Fiber-Reinforced Shell

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Nomenclature

a	= shell radius
A_{ij}	= elements of stretching stiffness matrix
B_{ij}	= elements of bending-stretching stiffness matrix
D_{ij}	= elements of bending stiffness matrix
$\bar{A}_{ij},\bar{B}_{ij},\bar{D}_{ij}$	$=\frac{1}{A_{22}}\left(A_{ij},\frac{B_{ij}}{a},\frac{D_{ij}}{a^2}\right)$
E_{11},E_{22},G_{12}	= orthotropic elastic constants
h	= thickness of shell
I_n	= modified Bessel function of first kind of order n
K_n	= modified Bessel function of second kind of order n
ℓ	= shell length
m,n	= number of axial half-waves and circumferential waves
p	= perturbation pressure
q_i	= surface loads on the shell
t	= time
u_1,u_2,u_3	= axial, circumferential and radial displacements
x,θ,r	= axial, circumferential and radial coordinates
$\bar{V}=V/(E_{11}/\rho_s)^{1/2}$	= dimensionless fluid velocity
α	$=x/a$
β	$=m\pi/\ell$
γ	= orientation of fiber axes relative to structural axes
ν	= major Poisson's ratio
δ_{ij}	= Kronecker delta
λ	$=m\pi a/\ell$
ρ_i,ρ_0	= mass density of fluid inside and outside of the shell
ω	= circular frequency
ω_0	$=(E_{11}/\rho_s a^2)^{1/2}$
Ω	$=\omega/\omega_0$
ϕ	= perturbation velocity potential
η_B,η_H	= parameters defined in Eq. (3)

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Introduction

VIBRATION characteristics of circular, cylindrical, isotropic shells, containing a flowing fluid, have been studied by several investigators using beam-type theory.[1] Sufficiently high flow velocity reduces the natural frequency of bending vibration and finally causes a static divergence instability. Static instability for short, thin, isotropic shells is quite often associated with higher-order circumferential modes; hence shell theory has to be utilized to predict the divergence instability for these modes. A number of authors (primarily Weaver and Paidoussis) have noted that flutter of a short, thin, cylindrical shell is possible for all end conditions, if the flow velocity is sufficiently high.

Within the last few years several papers have been published concerning the problem of an anisotropic circular cylindrical shell containing a flowing fluid.[2-5] This paper is based on the work presented in Ref. 3. An analytical method is developed to account for the effects of inviscid, incompressible, and irrotational flow on the vibration characteristics of thin, cylindrical fiber-reinforced shells. Particular attention is paid to the determination of the natural frequency of vibration and the divergence boundaries for various circumferential modes n. The results are derived by utilizing Galerkin's method to solve the Flügge-type equations of motion. Two cases are considered here: a shell containing a flowing fluid and a fluid conveying shell which is surrounded by a static fluid.

Shell Equations

A thin anisotropic cylindrical shell of length ℓ and thickness h conveys a fluid with velocity V. The shell consists of N homogeneous, orthotropic lamina. Each layer has arbitrary thickness, elastic properties, and orientation. A Flügge-type equation of motion is developed in cylindrical coordinates (x,θ,r) neglecting axial and circumferential inertia forces.[6]

$$\sum_{j=1}^{3} L_{ij}u_j=\delta_{i3}q_i \quad i=1,2,3 \tag{1}$$

where the differential operators are functions of $\bar{A}_{ij},\bar{B}_{ij},\bar{D}_{ij}$, α, and θ.

Equation (1) represents a linear system of equations in u_1,u_2, and u_3, which can be reduced by Gaussian elimination to one equation in terms of the radial displacement u_3:

$$[L_{11}^2L_{22}L_{33}-L_{11}L_{12}^2L_{33}-L_{13}^2L_{22}L_{11}-L_{23}^2L_{11}^2 + 2L_{23}L_{11}L_{12}L_{13}]u_3=[L_{22}L_{11}^2-L_{12}^2L_{11}]q_3 \tag{2}$$

where q_3 includes both the inertia of the shell and the dynamic pressure on the shell due to fluid flow.

Fluid Equations

The governing equation for the flow is Laplace's equation, written in terms of the perturbation velocity potential. The perturbation pressure p is then given by the linearized unsteady Bernoulli equation for the case of a fluid conveying